Lesson 150

Advanced Math

Accelerated Learning Lab

Variables

Lesson 150

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Variables

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Lesson 150 , Variables

CMA4150

To the Student

In Lesson 150, *Variables*, you will learn what variables are. You will also learn simple operations with variables.

In this book you will find numbered parts that are called Frames. Within these Frames, you will be asked to respond to many questions about variables. Write your answers to the questions on a separate piece of paper. Then look at the top of the next page in the book and check your answers.

If you use the lesson in this way, you will easily learn how to solve problems involving variables. If you do not understand how to use this book, or if you need assistance with certain Frames, ask your instructor for help.

Now turn the page and begin.

1. In this lesson you will learn what variables are. You will also learn to work simple problems with variables. ***Variables are symbols (usually letters) used to stand for unknown numbers***. A variable can be a letter, a Greek letter, or even a shape that stands for an unknown number. A number is never used as a variable, however.

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2. The letter ***a*** can be used as a _______ (geometric shape / variable). However, the number 2 would not be used as a variable.

---

3. The letter ***Z*** can be used as a _______. A variable can be written as an upper or lower case letter.

---

4. The Greek letter Ω can be used as a _______.

---

5. A variable _______ (can / cannot) be lower or upper case.

---

6. Write **yes** beside the letter if the symbol can be used as a variable. Write **no** beside the letter if the symbol cannot be used as a variable.

   A. B _______
   B. y _______
   C. 5 _______
   D. Θ(the Greek letter Theta) _______
   E. g _______
   F. 10 _______

answers to page 1

2. variable
3. variable
4. variable
5. can
6. A. yes
   B. yes
   C. no
   D. yes
   E. yes
   F. no

---

7. Since variables stand for numbers, you can add, subtract, multiply, and divide them just as you would numbers. When working problems with _______, you follow the same rules you follow when working with numbers.

---

8. Look at the example below.

   a + a = 2a

   You can add two variables together as long as they are the ***same*** variable. In the example above, the result is _______ (2 / 3) times the variable. When using variables, we usually write “two times a” as 2a; however, we could also write 2 • a.

---

9. Look at the following equations:

B + B + B = 3B	3B stands for “3 times B.”
x + x + x + x = 4x	_______ stands for “4 times x”
Θ + Θ + Θ + Θ + Θ = 5Θ	
n + n + n + n + n + n =	_______n
p + p + p + p + p =	_______

answers to page 2

7. variables
8. 2
9. 4x; 6; 5p

---

10. Complete the following equations:

A. g + g + g = _______

B. h + h + h + h + h + h = _______

C. r + r = _______

When you have a number times a variable, like 3g, the 3 is called the ***coefficient***. The coefficient is **the number in front of the variable**. 3g means the same as "multiply 3 by g."

---

11. Look at this example:

2a + 3a = 5a

When you add two expressions that contain the same variable:

Add the coefficients: 2 + 3 = _______.

Then, write the sum of the coefficients in front of the variable: _______a.

answers to page 3

10. A. 3g
    B. 6h
    C. 2r
11. 5; 5

---

12. A variable without a coefficient written in front of it has a coefficient of 1.

    n is the same thing as 1n

    x is the same thing as 1x

    m is the same thing as _______

    You will need to remember that variables written by themselves have 1 as a coefficient when adding coefficients and variables.

    For example,

    6r + r means the same as 6r + 1r.

    6r + 1r = 7r

    6r + r = 7r

    8x + x means the same as _______.

answers to page 4

12. 1m; $8x + 1x$ ***or*** $9x$

---

13. When adding coefficients with the same variable, add the coefficients, and write the coefficient in front of the variable.

$12b + 3b = 15b$

$6b + b = 7b$

You cannot add coefficients if the variables are not the same. For example,

$8t + 5r$ cannot be added

You also cannot add if the variables have different exponents. For example,

$5s + 6s^2$ cannot be added

Finish the equations below by adding the coefficients:

$3x^2 + 6x^2 =$ ________ x^2 (You can add the coefficients because the variables have the same exponent.)

$y + 10y =$ ________

---

14. Solve the following. To add variables with coefficients, the variables must be the same and have the same exponent. If you cannot add a problem, write **can't add** beside the letter.

A. $7y + y =$ ________

B. $8n^3 + 20n^3 =$ ________

C. $4s + 4g =$ ________

D. $27z + 8z =$ ________

answers to page 5

13. 9; 11y
14. A. 8y
    B. $28n^3$
    C. can't add (the variables are not the same)
    D. 35z

---

15. You can also add variables involving negative numbers. Follow the same rules you used above. Look at the example below:

    (-8a) + (-7a) = -15a

    Add the coefficients: (-8) + (-7) = _______.

    Write the sum of the coefficients in front of the variable: _______.

---

16. Work one more.

    6w + (-7w) =

    Add the coefficients: 6 + (-7) = _______.

    Write the sum of the coefficients in front of the variable: _______.

    A negative sign before a variable is the same as having -1 as a coefficient. Normally you do not write the 1.

    8x + (-9x) = -x

    10n + (-11n) = _______

answers to page 6

15. -15; -15a
16. -1; -1w ***or*** -w; -n

---

17. Find the sums of the following. If you can't add, write **can't add** beside the letter.

A. $(-9t^2) + (-5t^2) =$ ________

B. $(-6p) + (5p) =$ ________

C. $(-5w^2) + (4w^3) =$ ________

D. $(-b) + (2c) =$ ________

E. $17x + 13x =$ ________

---

18. To subtract variables, follow the same rules you used for addition. Subtract the coefficients instead of adding them. Look at the example below:

$a - a = 0$

Subtract the coefficients: 1 – 1 = _______.

Write the difference of the coefficients in front of the variable: 0_______.

When a variable has a coefficient of zero, the answer is zero.

answers to page 7

17. A. $-14t^2$
    B. -p (or -1p)
    C. can't add
    D. can't add
    E. 30x
18. 0; a

---

19. Work the next one.

    23d – 15d =

    Subtract the coefficients: 23 – 15 = _______.

    Write the difference of the coefficients in front of the variable: 8_______.

    The difference is _______.

---

20. Work one more.

    6x – (-5x) =

    Subtract the coefficients: 6 – (-5) = _______.

    Write the difference of the coefficients in front of the variable: 11_______.

    The difference is _______.

answers to page 8

19. 8; d; 8d
20. 11; x; 11x

---

21. Find the differences for the problems below. If you can't subtract, write **can't subtract** beside the letter.

A. $18y - 12y =$ _______

B. $\frac{1}{2}z - 6z =$ _______

C. $3a^2 - (-4a^2) =$ _______

---

22. Now, you will learn to multiply using variables. Look at the example below.

$a \bullet b = ab$

First, remember your multiplication signs. The expressions ***a • b*** and ***ab*** mean _______ (different / the same) thing(s): ***a times b***. The expression ***ab*** is a shorter way to write ***a • b***. When multiplying two different variables, you simplify the expression by writing the variables side by side.

---

23. Look at the following equation:

$2 \bullet z = 2z$

Numbers work the same as variables. You write the problem in a simplified way. The expressions *2 • z* and *2z* mean _______ (different / the same) thing(s).

answers to page 9

21. A. 6y
    B. $-5\frac{1}{2}z$
    C. $7a^2$
22. the same
23. the same

---

24. Simplify the following:

    A. g • h = _______

    B. 5 • c = _______

    C. y • z = _______

---

25. Look at the example below:

    2(3x) = 6x

    Here, you multiply the number outside the parentheses by the coefficient: 2 • 3 = _______.

    Write the product of the number and coefficient in front of the variable.

    The product is _______.

---

26. The product for the problem below is _______.

    8(2y) =

answers to page 10

24. A. gh
    B. 5c
    C. yz
25. 6; 6x
26. 16y

---

27. Placing variables side by side means the same as multiplying them. You place different variables together to signify multiplication. Look at the next example.

    a • 2 • b = 2ab

    Notice that the number is written first, and the letters _______ (are / are not) written in alphabetical order, making the solution easier to read.

    When you write a solution, write the number or coefficient first and the letters last. Write the letters in alphabetical order.

---

28. Remember to write the coefficient _______ (first / last). Then write the letters in alphabetical order.

    s • 5 • r = 5rs
    y • z • 7 = 7yz
    12 • w • v = _______
    h • g • (-4) = _______

answers to page 11

27. are
28. first; 12vw; -4gh

---

29. Find the products for the following:

A. a • b • 3 = _______

B. (-17) • q • p = _______

C. n • 10 • m = _______

---

30. Look at the example below. In a problem with variables, a number by itself, without a variable, is called a ***constant***.

4 • 5x • y = 20xy

In this example, multiply the constant, 4, by the coefficients, 5 and 1, and combine the variables. Then, write the number _______(first / second) and write the letters in alphabetical order.

---

31. Work this problem.

9b • 8c • a =

First, multiply the coefficients in the problem: 9 • 8 = _______.

Then, write the product of the problem in correct order. Remember to write the product of the numbers before the variables.

The product is _______.

answers to page 12

29. A. 3ab
    B. -17pq
    C. 10mn
30. first
31. 72; 72abc

---

32. Work one more.

$10y \cdot (-5x) \cdot (-z) =$

First, multiply the coefficients: 10(-5)(-1) = _______. Remember, a negative variable means the same as having a coefficient of -1. You must include the -1 when multiplying the numbers.

Then, write the product of the numbers before the letters.

The product is _______.

---

33. Find the products for the following.

A. $6t \cdot r \cdot 5s =$ _______

B. $(-b) \cdot 8c \cdot 6a =$ _______

C. $k \cdot 5l \cdot (-7j) =$ _______

answers to page 13

32. 50; 50xyz
33. A. 30rst
    B. -48abc
    C. -35jkl

---

34. So far, you have worked with different variables, and you have combined them when multiplying. When you multiply variables that are the same, you do something different. Look at the examples below:

$a \bullet a = a^2$
$a \bullet a \bullet a = a^3$
$a \bullet a \bullet a \bullet a = a^4$
$a \bullet a \bullet a \bullet a \bullet a =$ _______
$a \bullet a \bullet a \bullet a \bullet a \bullet a =$ _______

When you multiply numbers, you add the exponents, remember? Here, all of the exponents are 1. When you add the exponents, you get the exponent of the product. Then you write down the variable with the sum of the _______ (exponents / variables).

---

35. Look at the example below:

$6a \bullet 3a = 18a^2$

First, multiply the coefficients as you did when working with different variables: $6 \bullet 3 =$ _______.

Then multiply the variables: $a \bullet a =$ _______. (Remember, add the exponents.)

Now, write the product of the numbers in front of the product of the variables: _______.

answers to page 14

34. a^5, a^6; exponents
35. 18; a^2; $18a^2$

---

36. Work this problem.

$5y^2 \bullet 2y^3 =$

First, multiply the coefficients. _______

Then, multiply the variables. Remember, add the exponents. _______

The product is _______.

---

37. When multiplying with variables and exponents, you _______ the exponents in the product. Sometimes exponents may be negative. You will still add the exponents using what you've learned about adding positive and negative numbers.

For instance,

$x^4 \bullet x^{-2} = x^2 \quad (4 + (-2) = 2)$

$y^5 \bullet y^{-3} =$ _______

---

38. $n^{-4} \bullet n^2 =$ _______ A product can have a negative exponent.

answers to page 15

36. 10; y^5; $10y^5$
37. add; y^2
38. n^{-2}

---

39. A variable without an exponent written has an exponent of 1.

    x is the same thing as x^1

---

40. _______ the exponents when multiplying variables with exponents. Do not forget that a variable by itself has an exponent of _______.

    $x^{-4} \bullet x =$ _______

    $x^{-3} \bullet x =$ _______

---

41. $n^2 \bullet n^5 =$ _______

    $n \bullet n^4 =$ _______

answers to page 16

40. Add; 1; x^{-3}; x^{-2}
41. n^7; n^5

---

42. When doing more complicated multiplication with coefficients, variables, and exponents, you must remember the operations and pay attention to the signs.

$3x^5 \bullet x^{-2} =$

1. Multiply the coefficients. $3 \bullet 1 =$ _______
2. Add the exponents. $5 + -2 =$ _______
3. Combine coefficients, variables, and exponents. The product is $3x^3$.

---

43. $5x \bullet -3x^4 =$

The product of the coefficients is _______.

The product of the variables is _______.

$5x \bullet -3x^4 = -15x^5$

---

44. $n \bullet 4n^{-3} =$

The product of the coefficients is _______.

The product of the variables is _______.

$n \bullet 4n^{-3} = 4n^{-2}$

answers to page 17

42. 3; 3
43. -15; x^5
44. 4; n^{-2}

---

45. $5n \bullet 3n =$

The product of the coefficients is _______.

The product of the variables is _______.

$5n \bullet 3n =$ _______

---

46. A negative coefficient multiplied by another negative coefficient yields a _______ product.

$-x \bullet -3x^3$

$-1 \bullet -3 = 3$	multiply coefficients
$x \bullet x^3 = x^4$	multiply variables
$-x \bullet -3x^3 = 3x^4$	combine for the product

---

47. $-3y^4 \bullet -5y^3 =$ _______

---

48. $-2n^2 \bullet -3n^{-4} =$ _______

---

49. $3x^{-4} \bullet x =$ _______

answers to page 18

45. 15; n^2; $15n^2$
46. positive
47. $15y^7$
48. $6n^{-2}$
49. $3x^{-3}$

---

50. A variable without a written exponent has an exponent of _______.

A variable without a written coefficient has a coefficient of _______.

$x \bullet x^2 = x^1 \bullet x^2 = x^3$

$-n \bullet 3n^2 = -1n^1 \bullet 3n^2 = -3n^3$

---

51. Work one more.

$17y^3 \bullet (-2y^{-5}) =$

First, multiply the coefficients. _______

Then, multiply the variables. _______

The product is _______.

---

52. Find the products for the following.

A. $a \bullet 6a^{-3} =$ _______

B. $7z^5 \bullet 8z^4 =$ _______

C. $2r^2 \bullet 3r^2 =$ _______

answers to page 19

50. 1; 1
51. -34; y^{-2}; $-34y^{-2}$
52. A. $6a^{-2}$
    B. $56z^9$
    C. $6r^4$

---

53. Find the products for the following.

A. $3(-2r^2) =$ ________

B. $6a \bullet \frac{1}{2} c \bullet b =$ ________

C. $4y \bullet 6y^3 =$ ________

---

54. Good work! Now you will learn to divide variables. First, you need to remember the following rules.

$$\frac{a}{a} = 1$$

$$-\frac{a}{a} = -1$$

$$\frac{-a}{-b} = \frac{a}{b}$$

$$\frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$$

$$(-1)\frac{a}{b} = -\frac{a}{b}$$

answers to page 20

53. A. $-6r^2$
B. 3abc
C. $24y^4$

---

55. Use the rules in Frame 54 to complete the following:

The expression $-\frac{x}{x}$ is equal to _______.

The expression $\frac{-y}{z}$ equals _______ or _______.

The expression $\frac{-h}{-j}$ is the same as _______.

---

56. Now you will learn to work division problems involving variables. Look at the example below:

$a^4 \div a^2 = a^2$

When you divide with variables and exponents, you subtract the exponents. Then write the variable with the _______ (difference / product) of the exponents.

---

57. $x^4 \div x^2 = x^2$

$y^{10} \div y^7 =$ _______

$n^3 \div n =$ _______

You can only subtract exponents if the variables are the same.

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End of Lesson Sample

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